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The Reflecting Beam Waveguide

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Summary—**In this paper a type of beam waveguide which uses appropriately shaped metal reflectors instead of dielectric lenses as the phase correcting devices is described. A theory has been developed which, subject to certain restrictions, describes the modes of this type of beam waveguide and predicts a loss of the order of of 0.01 db per iteration.**

A reflecting beam waveguide comprising eight aluminum reflectors has been investigated at a wavelength of 4 millimeters. The measured loss per iteration is approximately 0.015 db which is in good agreement with the theoretical value. The cross-sectional electric field distribution has also been measured and found to be in satisfactory agreement with the theory.

It is shown that the reflecting beam waveguide is a practical system for the transmission of power at submillimeter wavelengths.

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Introduction

IN RECENT YEARS, the beam waveguide analyzed by Goubau and his co-workers [1], [2] has been considered as a possible transmission system for wavelengths from the low millimeter to the optical region of the spectrum. Basically, the beam waveguide consists of dielectric lenses or phase correctors, each having a focal length f, which are separated by a distance $a = 2f$ as shown in Fig. 1. At each lens the phase distribution of a signal propagating along the beam waveguide is corrected to compensate for diffraction effects due to the finite aperture of the beam waveguide. The confocal spacing keeps these diffraction losses at a minimum for a given lens diameter. The losses on this type of waveguide are due to the diffraction losses cited above, to absorption losses within the lenses, and to reflection and scatter losses at the surfaces of the lenses.

For reasonable lens diameters and spacings the diffraction losses can be kept quite small in the millimeter and submillimeter region of the spectrum. However, it is difficult to obtain a dielectric phase corrector for which both the absorption and reflection losses are small at

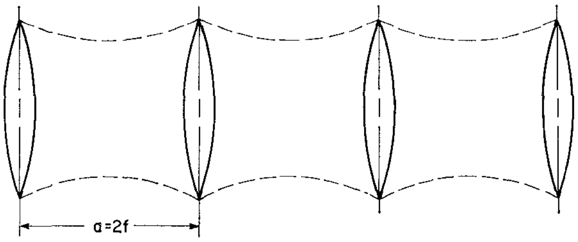
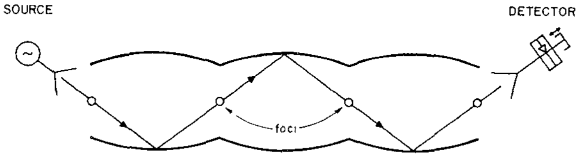
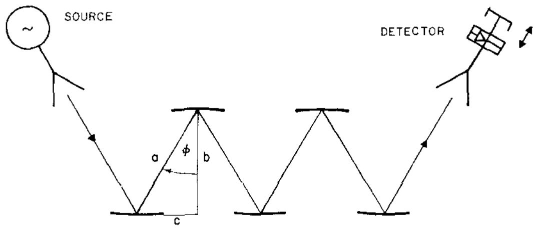


Fig. 1—Dielectric lens beam waveguide.



(a)



(b)

Fig. 2—Reflecting transmission systems.

these wavelengths. These losses associated with the dielectric lenses may be avoided if metal reflectors are used to guide the wave beam.

Earlier work by Boyd and Gordon [3] and Fox and Li [4] on confocal resonators and maser interferometers suggests that such a possibility exists. Damon and Chang [5] have proposed such a reflecting beam waveguide. Their transmission line, based on the principles of geometrical optics, comprised a colinear series of spheroidal reflectors with common foci as shown in Fig. 2(a). However, Boyd and Kogelnik [6] have recently shown that this arrangement (which corresponds to a spacing $a=4f$ for a lens type beam waveguide) has very high diffraction losses and thus is unsatisfactory for a transmission line of any appreciable length.

In this paper a reflecting beam waveguide, shown schematically in Fig. 2(b), in which elliptic paraboloidal reflectors are used as the phase correcting elements is described. An analysis is presented which, subject to certain restrictions, describes the modes on this type of transmission line and predicts a fractional loss per iteration of the order of 0.01 db. Experimental data for both the field distribution for lowest order mode transmission and for the losses on the reflecting beam waveguide are in substantial agreement with the theory.

Analysis

In Fig. 2(b), a cross-sectional view of the reflecting beam waveguide in which the reflectors are represented by their traces in the plane of the figure is shown. The orientation of the reflectors may be described by any two of the parameters a, b, c, and ϕ.

Fig. 3 is a three-dimensional view of a reflector and the elliptic paraboloid of which it is a part. The length of the reflector (measured in the direction of the y_1 axis) is $2sa/b$, and its width (measured in the direction of the x_1 axis) is $2s$. The equation of the reflector surface is chosen to be

$$z_1 = \frac{x_1^2}{2b} + \frac{by_1^2}{2a^2} \tag{1}$$

for reasons which will become apparent as the analysis progresses. It should be pointed out that only an approximate analysis is presented in this paper. A slightly different formulation of the problem which is not quite as accurate as the analysis presented here indicates that the surface

$$z_1 = \frac{bx_1^2}{2a^2} + \frac{b^3y_1^2}{2a^4}$$

is also consistent with the approximations made in this paper [7]. The experimental performance of both types of reflector is comparable, which is not too surprising in view of the work of Boyd and Kogelnik on generalized confocal resonators [6].

From both of the above equations it follows that the reflector shape also depends on the parameters a and b, and in the limit when $a=b$; *i.e.*, when $\phi=0$, the reflectors become paraboloids of revolution and the reflecting beam waveguide folds into a confocal resonator.[1] Based upon this observation, it seemed reasonable to suppose that the analysis used by Boyd and Gordon [3] to describe the confocal resonator might be adapted to describe the reflecting beam waveguide as well. This supposition was in fact valid, and the analysis developed below for the reflecting beam waveguide follows closely the work of Boyd and Gordon [3]. Only the surface given by (1) will be considered in detail here.

For the analysis one considers a system comprising two reflectors from the waveguide as shown schematically in Fig. 4. It is assumed that the reflector dimensions are small compared to the spacing a and that the reflector dimensions are large compared to a wavelength. It is also assumed that the electric field on the surface P', E_x' (x', ξ'), is linearly polarized in the x direction. A scalar formulation of Huygens' principle may then be used to determine the field at the surface of the right-hand reflector, $E_x(x, \xi)$.

$$E_x(x, \xi) = \iint_{S'} \frac{ik(\cos\theta + \cos\psi)}{4\pi\rho} \epsilon^{-ik\rho} E_x'(x', \xi') dS' \tag{2}$$

[1] As noted by Boyd and Gordon [3] in discussing the confocal resonator, one cannot distinguish between parabolic and spherical reflectors if the reflector dimensions are sufficiently small. Thus only the radii of curvature in the x_1 and y_1 directions are the important parameters.

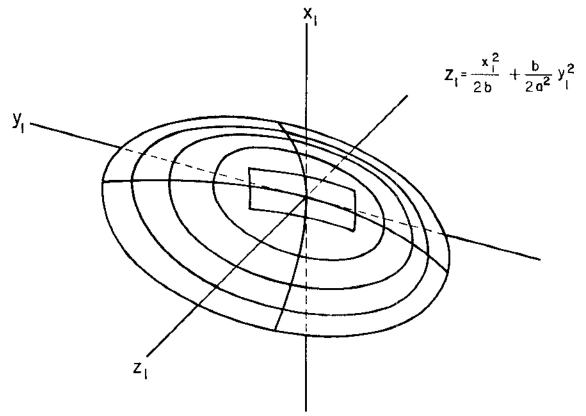


Fig. 3—Three-dimensional view of a reflector and the elliptic paraboloid of which it is a part.

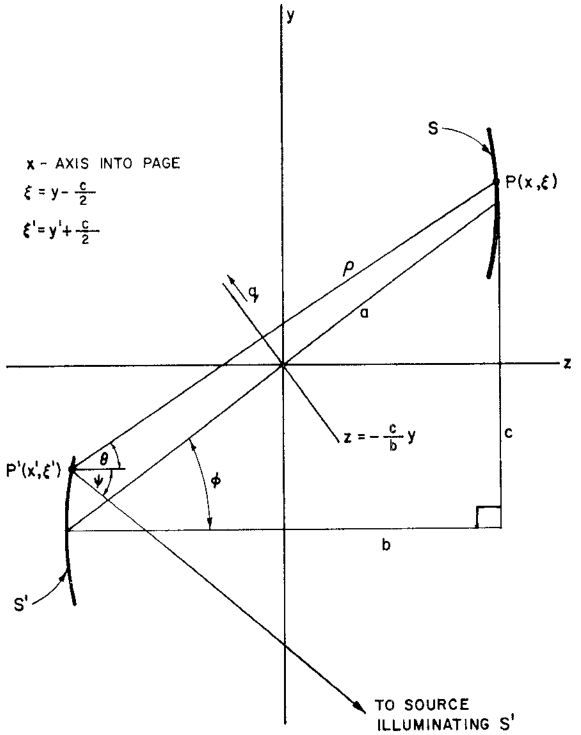


Fig. 4—Geometry of the reflecting beam waveguide.

where

ρ = distance between P' and P
$= [(x-x')^2+(\xi-\xi'+c)^2+(z-z')^2]^{\frac{1}{2}}$

θ = angle between the normal to the reflector surface at P' and the line $P'P$

ψ = angle between the normal to the reflector surface at P' and the line from P' to the source illuminating S'

$\lambda = 2\pi/k$ = wavelength.

In this far field approximation, θ and ψ are considered to be essentially constant and define the ray trajectory which results when the phase front across the aperture is very nearly planar, though not necessarily parallel to the aperture.

One might note that the integral equation usually associated with Huygens' principle is

$$E_x(x, \xi) = \iint_{S'} \frac{ik(1+\cos\theta)}{4\pi\rho} \epsilon^{-ik\rho} E_x'(x', \xi') dS'.$$

This equation is valid, however, only when the surface S' is an equiphase surface. For the more general situation when S' is not an equiphase surface, (2) must be used [8]. In this case, however, because the reflector curvature is small and the reflector dimensions are small in comparison to the spacing a, it follows that $\theta \cong \phi$. Furthermore, because the source producing the field distribution on the left-hand reflector (S') is just another reflector of the waveguide, $\psi \cong \phi$. Consequently, (2) reduces to

$$E_x(x, \xi) \cong \iint_{S'} \frac{ik\cos\phi}{2\pi\rho} \epsilon^{-ik\rho} E_x'(x', \xi') dS'. \quad (3)$$

The equations for the reflector surfaces in the coordinate system of Fig. 4 can be obtained from (1) which represents the reflector surface in the coordinate system of Fig. 3 by a straightforward coordinate transformation. The resulting equation for the reflector on the right is

$$z = \frac{b}{2} - \frac{x^2}{2b} - \frac{b}{2a^2}\xi^2$$

where

$$\xi = y - \frac{c}{2}\cdot$$

Similarly, for the reflector on the left,

$$z' = -\frac{b}{2} + \frac{x'^2}{2b} + \frac{b}{2a^2}\xi'^2$$

where

$$\xi' = y' + \frac{c}{2}\cdot$$

From these equations ρ, the distance from a point $P'(x', \xi')$ on the left reflector to a point $P(x, \xi)$ on the right reflector, can be shown to be

$$\rho = \left\{(x-x')^2 + (\xi - \xi' + c)^2 + \left[b - \frac{1}{2b}(x^2+x'^2) - \frac{b}{2a^2}(\xi^2+\xi'^2)\right]^2\right\}^{1/2}.$$

With the aid of the binomial theorem, it follows that

$$\rho = a\left\{1 + \frac{c}{a^2}(\xi - \xi') - \frac{xx'}{a^2} - \frac{b^2}{a^4}\xi\xi' + \cdots\right\}. \quad (4)$$

The higher order terms in (4) make a negligible contribution to the phase term $\epsilon^{-ik\rho}$ of (3) provided

$$\frac{s^2}{a\lambda} \ll \frac{ab}{2cs[1+(a/b)^2]}. \quad (5)$$

To facilitate solution of the integral equation (3) an assumption regarding the functional form of $E_x'(x', \xi')$ is now made. Because of the similarity between this system and the confocal resonator, one is led to assume a field distribution which is similar to that of the confocal resonator except for the addition of a phase term due to the fact that the incident beam of radiation impinges upon the reflectors at an angle of incidence approximately equal to ϕ. If $E_x'(x', \xi')$ is assumed to be of the form

$$E_x'(x', \xi') = E_0 f_m(x') g_n(\xi') \epsilon^{-ikc\xi'/a},$$

where E_0 is a constant and $f_m(x')$ and $g_n(\xi')$ are functions of x' only and ξ' only, respectively, then, (3) becomes

$$E_x(x, \xi) = \iint_{S'} \frac{ik \cos\phi}{2\pi\rho} \epsilon^{-ik\rho} E_0 f_m(x') g_n(\xi') \epsilon^{-ikc\xi'/a} dS'. \quad (6)$$

Because the apertures are assumed to be small compared to the reflector spacing a, ρ may be replaced by a except in the phase factor of (6). The differential area dS' is given by

$$dS' \cong dx' d\xi'$$

because the curvature of the reflectors is small. Substitution of the values for ρ, ϕ and dS' into (6) yields

$$E_x(x, \xi) = \int_{-as/b}^{as/b}\int_{-s}^{s} \frac{ikb}{2\pi a^2} \epsilon^{-ik[a+(c/a)\xi-(xx'/a)-(b^2/a^3)\xi\xi']} \cdot E_0 f_m(x') g_n(\xi') dx' d\xi'. \quad (7)$$

The eigenfunctions or normal modes of the reflecting beam waveguide are determined by the requirement that $E_x(x, \xi)$ be the same as $E_x'(x', \xi')$ except for a multiplicative constant; *i.e.*, it is required that

$$E_x(x, \xi) = \sigma_m \sigma_n E_0 f_m(x) g_n(\xi) \epsilon^{-ikc\xi/a}$$

where σ_m and σ_n are constants. Substitution of this expression in (7) yields the following integral equation:

$$\sigma_m\sigma_n f_m(x) g_n(\xi) = \frac{ikb}{2\pi a^2} \epsilon^{-ika} \int_{-as/b}^{as/b}\int_{-s}^{s} \epsilon^{(ik/a)[xx'+(b/a)^2\xi\xi']} \cdot f_m(x') g_n(\xi') dx' d\xi'. \quad (8)$$

It is convenient to introduce a transformation of variables at this point. Let

$$d = s^2 k/a$$

$$X = x\sqrt{k/a} = \frac{x}{s}\sqrt{d}$$

$$\zeta = \xi \frac{b}{a}\sqrt{k/a} = \frac{\xi}{s}\frac{b}{a}\sqrt{d}$$

$$F_m(X) = f_m(x)$$

$$G_n(\zeta) = g_n(\xi).$$

Then (8) becomes

$$F_m(X)G_n(\zeta) = \frac{i\epsilon^{-ika}}{2\pi\sigma_m\sigma_n}\int_{-\sqrt{d}}^{\sqrt{d}} F_m(X')\epsilon^{iXX'}dX' \cdot \int_{-\sqrt{d}}^{\sqrt{d}} G_n(\zeta')\epsilon^{i\zeta\zeta'}d\zeta' \quad (9)$$

which may be separated into the two identical integral equations

$$F_m(X) = \frac{1}{\sqrt{2\pi}\,\Omega_m}\int_{-\sqrt{d}}^{\sqrt{d}} F_m(X')\epsilon^{iXX'}dX' \quad (10)$$

and

$$G_n(\zeta) = \frac{1}{\sqrt{2\pi}\,\Omega_n}\int_{-\sqrt{d}}^{\sqrt{d}} G_n(\zeta')\epsilon^{i\zeta\zeta'}d\zeta' \quad (11)$$

where Ω_m and Ω_n are constants defined by

$$\Omega_m\Omega_n i\epsilon^{-ika} = \sigma_m\sigma_n. \quad (12)$$

Eqs. (10) and (11) are the same as those obtained by Boyd and Gordon [3] in their analysis of the confocal resonator, and it is for this reason that the reflector surface given by (1) was chosen. It follows, therefore, that the modes of the reflecting beam waveguide are the same as those of the confocal resonator except for the phase shift along the surface of the reflectors. A complete discussion of the modes is given by Boyd and Gordon. In the following discussion only the lowest order mode will be considered because it has the lowest loss and is most suitable for a transmission system. The field distribution for the lowest order mode on the reflector is given by

$$\boldsymbol{E} = \boldsymbol{i}E_x = \boldsymbol{i}E_0\epsilon^{-(k/2a)[x^2+(b^2\xi^2/a^2)]}\epsilon^{-(ikc/a)\xi} \quad (13)$$

$$\boldsymbol{H} = \left(\frac{b}{a}\boldsymbol{j} - \frac{c}{a}\boldsymbol{k}\right)\sqrt{\epsilon_0/\mu_0}\,E_x \quad (14)$$

where the geometrical configuration is shown in Fig. 4, $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are unit vectors in the x, y, and z directions, respectively; ϵ_0 and μ_0 are the permittivity and permeability of free space, respectively; and $\boldsymbol{H}$ is the magnetic field intensity.

The electric field distribution in the plane mid-way between the reflectors [$y=-(b/c)z$ in Fig. 4] may be found by inserting the expression given in (13) for the field distribution on the reflector surface into (3), where ρ is now the distance from the point $P'(x', \xi')$ on the reflector surface to the point $P[x, y, z=-(c/b)y]$. In a straightforward way [3] one obtains

$$E_x(x, y) \cong \sqrt{2}\, E_0 \epsilon^{i((\pi/4)-(ka/2))} \epsilon^{-(k/a)(x^2+(a^2/b^2)y^2)}. \quad (15)$$

If q is a coordinate measured in the mid-plane $y=-(c/b)z$ perpendicular to the x axis as shown in Fig. 4, then $q=(a/b)y$. In terms of the variable q, (15) becomes

$$E_x\left(x, \frac{b}{a} q\right) \cong \sqrt{2}\, E_0 \epsilon^{i[(\pi/4)-(ka/2)]} \epsilon^{-(k/a)(x^2+q^2)}. \quad (16)$$

One may conclude that the mid-plane is an equiphase surface and that the field amplitude is given by a Gaussian function of the distance from the beam axis; *i.e.*, the line joining the reflector centers.

From (9)–(12) it follows that the power lost per reflection due to diffraction losses P_{LD} is given by

$$P_{LD} = \{1 - |\Omega_m \Omega_n|^2\} P_T \quad (17)$$

where P_T is the transmitted power. The dependence of the diffraction losses for the lowest order mode on the reflector size is shown in Fig. 5 in which P_{LD}/P_T is plotted as a function of the parameter $s^2/(a\lambda)$. To keep the diffraction losses low, the parameter $s^2/(a\lambda)$ should be at least of the order of 0.75.

In addition to diffraction losses there are also conduction losses due to the finite conductivity of the metal reflectors. These losses are a function of the field polarization and the reflector spacing as well. The power lost per iteration (or reflection) due to conduction losses P_{LW} is given by

$$P_{LW}/P_T = 4R_s \frac{\iint |H_t|^2 dS}{\mathrm{Re} \iint \mathbf{E} \times \mathbf{H}^* \cdot d\mathbf{S}} \quad (18)$$

where

$$R_s = \sqrt{\pi \mu f/\sigma}$$

the surface resistance, is a function of the permeability of the surface μ, the frequency f, and the conductivity σ. The integration extends over the reflector surface and H_t is the tangential component of the magnetic field intensity in the incident wave.

If the electric field is in the x direction as shown in Fig. 4, (18) becomes

$$P_{LW}/P_T = 4R_s \frac{\iint \left|\frac{b}{a} \mathbf{H}\right|^2 dxdy}{\sqrt{\mu_0/\epsilon_0}\left(\frac{b}{a}\right) \iint |\mathbf{H}|^2 dxdy}$$
$$= 4(b/a)\sqrt{\pi \epsilon_0 f/\sigma}.$$

If the magnetic field is in the x direction, the fractional conduction loss is given by

$$P_{LW}/P_T = 4R_s \frac{\iint |\mathbf{H}|^2 dxdy}{\sqrt{\mu_0/\epsilon_0}\left(\frac{b}{a}\right) \iint |\mathbf{H}|^2 dxdy}$$
$$= 4(a/b)\sqrt{\pi \epsilon_0 f/\sigma}.$$

Examination of these results shows that the former polarization is preferable because $b < a$.

Design and Fabrication of a Reflecting Beam Waveguide

In designing a low-loss reflecting beam waveguide, several factors must be considered. First, the reflectors must be large enough to keep diffraction losses low and yet small enough so that any higher-order modes generated by either the launching horn or imperfections or obstacles along the waveguide are rapidly attenuated. Both of these conditions are satisfied if $0.8 \leq s^2/(a\lambda) \leq 1$. Second, the analysis imposes the restriction

$$s^2/(a\lambda) \ll \frac{ab}{2cs[1 + (a/b)^2]}.$$

In practice, this restriction can be relaxed somewhat for the lower-order modes. In fact, for the test set-up described below,

$$s^2/(a\lambda) \cong ab/(24cs)$$

due to space limitations in the laboratory, and the experimental results are still in essential agreement with the theory. Third, the electric field should be polarized perpendicular to the plane of incidence to minimize conduction losses.

With these requirements in mind, an experimental reflecting beam waveguide was built having the following parameters:

$$a = 28.3 \text{ cm}$$
$$b = 20 \text{ cm}$$
$$c = 20 \text{ cm}$$
$$\phi = 45^\circ$$
$$s = 2.93 \text{ cm}$$
$$= 73.2 \text{ Gc}.$$

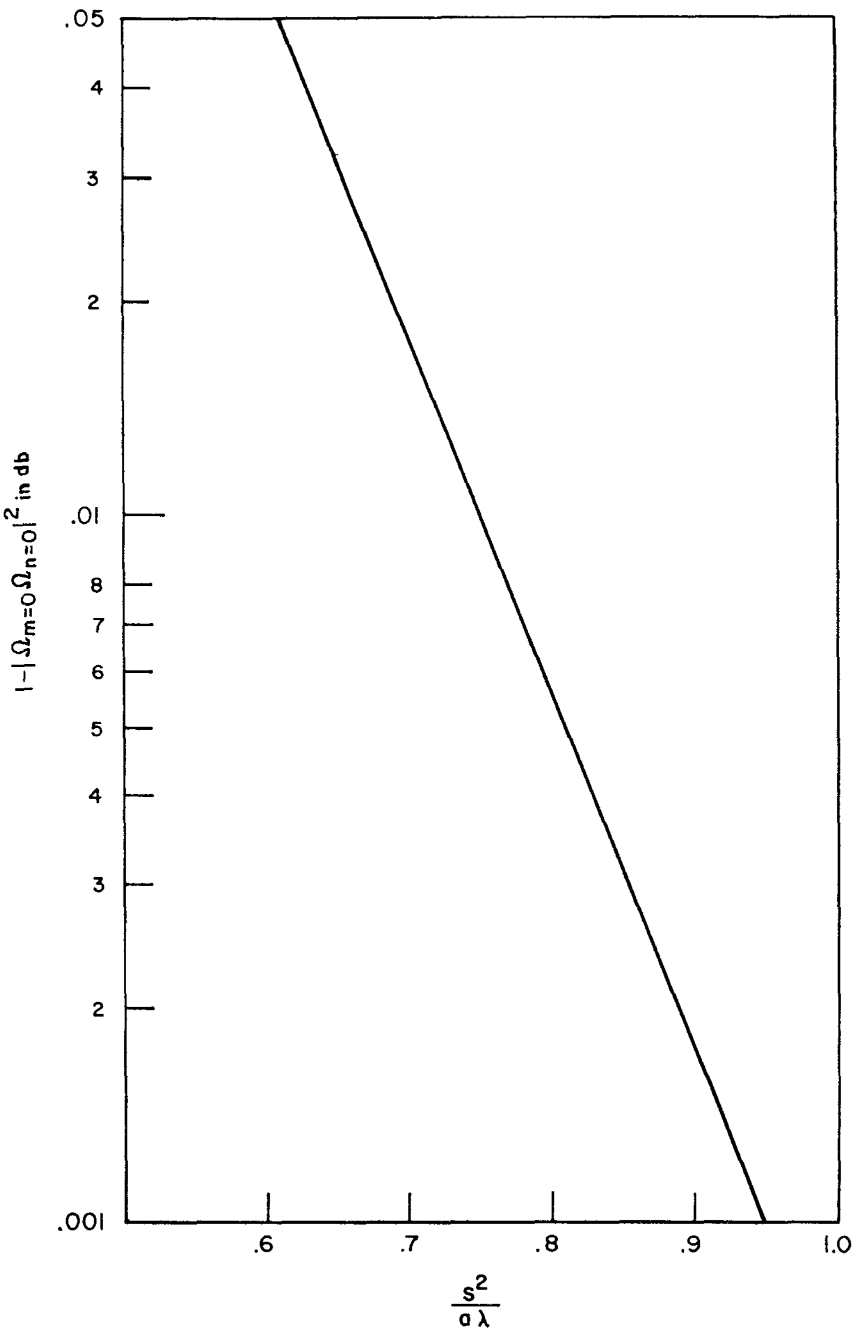


Fig. 5—Diffraction loss per iteration for lowest order mode as a function of $s^2/(a\lambda)$.

Because the shape of the reflectors prevents them from being made on a conventional lathe or milling machine, a technique was developed [9] using a vacuum to form a stretched aluminum membrane into the proper shape. It should be pointed out that in the analysis the reflectors were assumed to be rectangular, whereas an elliptical boundary is necessary for proper curvature of the aluminum membrane. For the experimental line, the calculated rectangular reflector dimensions are 5.86 cm by 8.29 cm. The reflectors were made large enough to enclose the calculated rectangle as shown in Fig. 6. This change in reflector shape does not affect the field distribution for the lowest order mode but does tend to lower the diffraction losses slightly.

A completed elliptic paraboloidal reflector is shown in Fig. 7, and the completed experimental reflecting beam waveguide is shown in Fig. 8.

Experimental Measurements

The cross-sectional field distribution measurements were made using a small 4-mm horn connected to a bolometer to probe the fields. The horn and bolometer were mounted on a carriage from a jeweler's lathe so the position of the horn could be accurately measured.

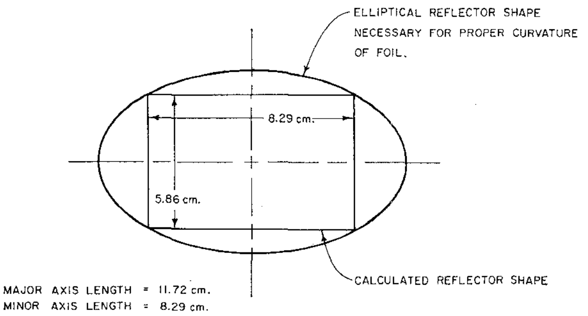


Fig. 6—Theoretical and experimental reflector shape.

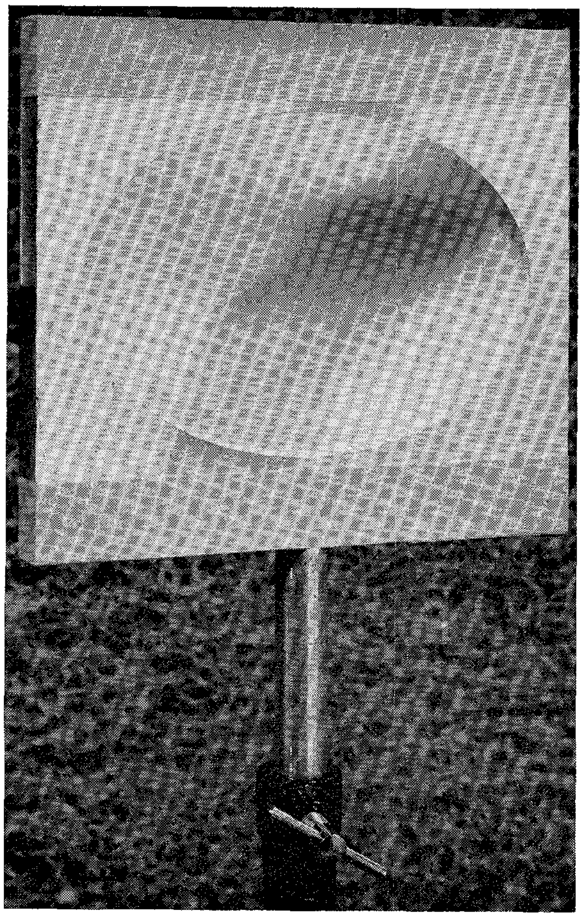


Fig. 7—Elliptic paraboloidal reflector.

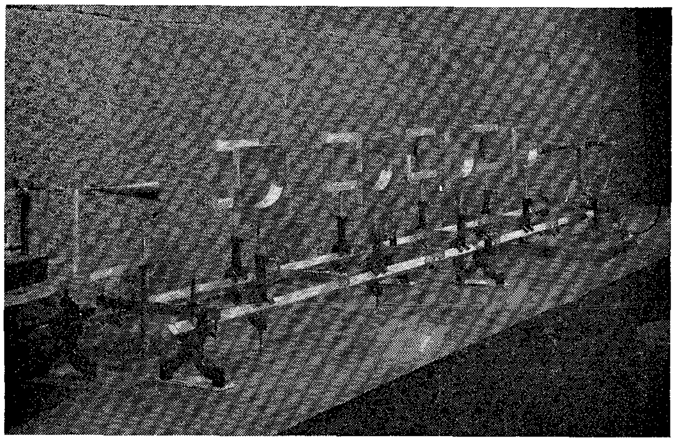


Fig. 8—Laboratory model of the reflecting beam waveguide.

The field patterns were measured in the mid-plane between two reflectors.

Fig. 9 shows a comparison between theoretical and experimental cross-sectional field distributions. In the plane halfway between the first and second reflectors (—○— curve), the field distribution shows the presence of higher order modes. These modes arise because the electric field distribution in the launching horn differs from the lowest order mode distribution of the reflecting beam waveguide. These higher order modes attenuate rapidly, however, and in the mid-plane between the seventh and eighth reflectors (—△— curve), the field distribution is very close to the theoretical value.

In order to investigate the losses of the reflecting beam waveguide, a resonator consisting of $4\frac{1}{2}$ sections of the waveguide was set up as shown in Fig. 10. To couple power into the resonator, a flat grating coupler [10] was placed at a distance $a/2$ in front of the first reflector. To terminate the resonator, a spherical reflector was placed at a distance $z_n = a$ from the last waveguide reflector. An alternate method of terminating the resonator would have been to place a flat reflector at a distance $z_n = a/2$ behind the last waveguide reflector. For this resonator α, the fractional loss per iteration, is given approximately by

$$\alpha \cong \frac{2\pi a}{Q_u \lambda} \tag{19}$$

where

a = distance between reflectors as shown in Fig. 10

Q_u = unloaded Q of the resonator

λ = wavelength.

For the case when the electric field is polarized normal to the plane of incidence in Fig. 10, the general shape of the resonance curve is shown in Fig. 11(a). To calibrate the horizontal scale of the oscilloscope trace in Mc/cm the frequency meter pip was first placed 1 cm to the left of the resonance and then 1 cm to the right of the resonance as shown in Figs. 11(b) and (c). The difference in readings on the frequency meter corresponded to a scale calibration of 9.8 Mc/cm. The oscilloscope pattern was then expanded horizontally by a factor of five to yield the trace shown in Fig. 11(d). From this trace the half power difference frequency was determined to be approximately 1 Mc. This gives a value of 73,200 for Q_L, the loaded quality factor. Q_u is related to Q_L by the relation

$$Q_u = 2Q_L\left(1 - \frac{\Gamma_t}{1 - \Gamma_t}\right)$$

where Γ_t is the reflection coefficient of the resonator at resonance. From the oscilloscope trace in Fig. 11(a),

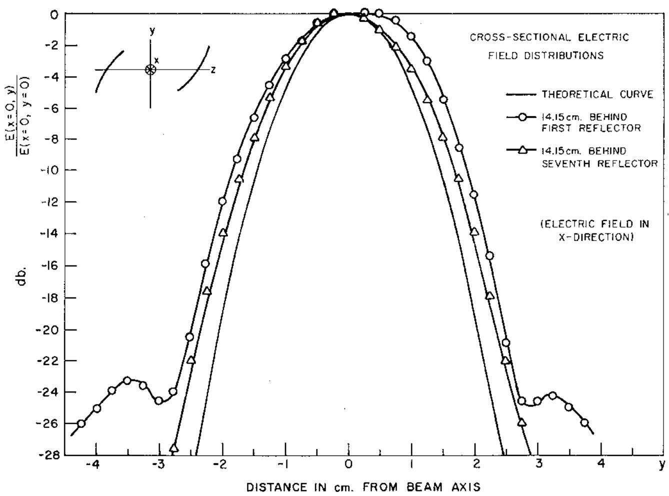


Fig. 9—Cross-sectional electric field distributions.

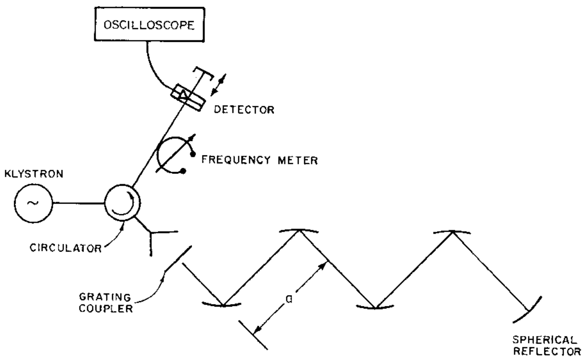


Fig. 10—Resonator for making loss measurements.

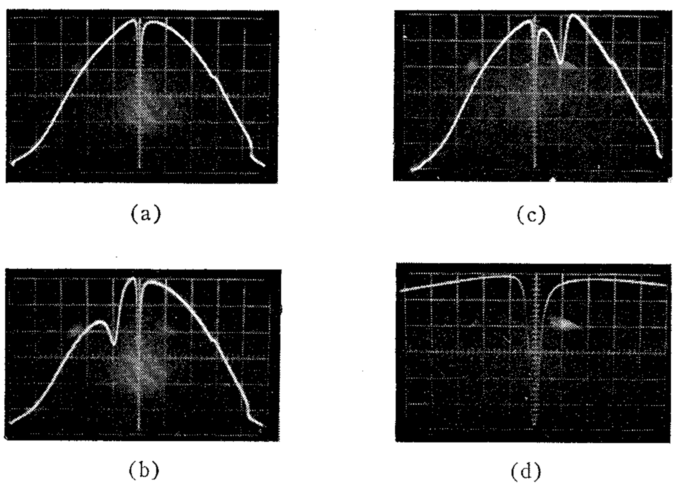


Fig. 11—Oscilloscope traces used in making Q measurements.

Γ_t can be determined to be $\sqrt{1/30}$ or 0.182 giving a value for Q_u of

$$Q_u = 2Q_L(0.88) = 129{,}000.$$

Substitution of this value of Q_u into (19) yields a value for α of 0.015 db per iteration which is in satisfactory agreement with the theoretical value of approximately 0.013 db per iteration.

If the launching horn is rotated 90° about its axis so that the electric field lies in the plane of incidence, the theory predicts that the conduction losses double and the over-all losses increase by approximately 25 per cent. Experimentally, α is found to increase by approximately 32 per cent to 0.02 db per iteration.

Work is now underway to develop components compatible with the reflecting beam waveguide. Most of the components can be modeled after their optical counterparts. For example, bends can be made from reflecting sheets, attenuators from coupled prisms, and an impedance measurement device can be made from a Michelson interferometer. Frequency measurements can be made with a confocal resonator coupled to the lineby a grating coupler.

Conclusions

The reflecting beam waveguide is a very practical transmission system for millimeter and submillimeter waves. The physical dimensions of the reflecting beam waveguide are large compared to the wavelength, whereas a conventional waveguide system has transverse dimensions which are of the order of one-half wavelength. In addition the power handling capability is higher and the attenuation lower for the reflecting beam waveguide than for conventional waveguides.

Although the lens beam waveguide has similar advantages with respect to a conventional waveguide, two problems must be considered if one wishes to use a lens beam waveguide at submillimeter wavelengths. One of these is associated with a restriction on the parameter $s^2/(a\lambda)$. To minimize the losses and to insure that higher-order modes attenuate rapidly, the value of $s^2/(a\lambda)$ should be approximately 0.8, the exact value depending on the properties of the lens material. Consequently, at shorter wavelengths the lenses become extremely small and difficult to fabricate for reasonable values of a, the distance between the lenses. One solution to this problem is to make oversize lenses and to use irises to restrict the aperture for the purpose of suppressing higher-order modes. Unfortunately, because the oversize lenses are thicker than lenses of the correct size, the dielectric losses for such a waveguide are greater than for a waveguide with lenses of the correct size. For the reflecting system, the restriction on $s^2/(a\lambda)$ is essentially the same; *i.e.*, $s^2/(a\lambda) \cong 0.8$. In this case, however, the technique of using oversize reflectors in conjunction with irises to restrict the aperture may be used with no increase in the losses as compared to a waveguide using reflectors of the correct size.

A second difficulty with the lens beam waveguide arises from the losses due to reflection at the lens surfaces and to the nonzero loss tangent of the dielectric material from which the lenses are fabricated. For presently available materials these losses become excessively large for wavelengths less than a few millimeters. For the reflecting beam waveguide, the losses due to the correcting elements arise only from the finite conductivity of the reflecting surfaces. Even at wavelengths as short as 0.1 millimeter, the calculated conduction losses are only approximately 0.02 db per iteration for aluminum reflectors, making the reflecting beam waveguide usable well into the submillimeter wave region.

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Analysis of a Differential Phase Shifter

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Summary—**This paper presents the theory and analysis of a ganged pair of "line stretcher" microwave phase shifters. The error analysis shows that some of the errors inherent in a single phase shifter of this type can be reduced through the use of a differential system; however, the magnitudes of other errors may more than offset the reduction. Graphical data are included to facilitate the rapid determination of the limit of error for any specified angle measurement.**

Introduction

An ERROR analysis by Schafer and Beatty of the reflectometer type phase shifter proposed by Magid leads to the conclusion that the accuracy of the device is limited by errors in the determination of the guide wavelength and position of the sliding short. The following paper presents an analysis of a phase shifter consisting of two ganged shifters of the Magid type in tandem; that is, a differential phase shifter, as proposed by Beatty.

A desirable characteristic of the differential phase shifter when compared to the Magid type is a reduction of the error that is introduced by short circuit displacement measurement tolerances. This paper contains graphical data for WR-90 and WR-112 waveguides so it can readily be determined which type of phase shifter has the least limit of error for a given measurement. A comparison example will be included.

The sources of error which are considered include those introduced by reflectometer tuning imperfections, waveguide width tolerances, short circuit displacement measurement and short circuit misalignment errors. Only these errors are considered because they limit the over-all accuracy that can be attained with either phase shifter.

Theory

Magid[1] proposed a phase shifter consisting of a directional coupler, matching transformers and precision waveguide section terminated in a sliding short circuit, as shown in Fig. 1. Assuming that $\Gamma_{2i}=0$ and $S_{31}=0$, the change of phase of the emerging signal b_3 is exactly equal

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